6.1

$$f(x)=\sum_{n=0}^{\infty} a_n(x-c)^n$$

$0^0=1$ BY DEF'N (FOR POWER SERIES)

$$f'(x)=\sum_{n=\cancel{0}1}^{\infty} n a_n (x-c)^{n-1}=\sum_{n=0}^{\infty}(n+1)a_{n+1}(x-c)^n$$

$$f''(x)=\sum_{n=\cancel{1}2}^{\infty} n(n-1)a_n(x-c)^{n-2}=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-c)^n$$

IF $c=0$

$$f(x)=\sum_{n=0}^{\infty} a_n x^n \qquad f'(x)=\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n \text{ OR } \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$f''(x)=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$$

$$\text{OR } \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

IF $f(x)=\sum_{n=0}^{\infty} a_n x^n$

$$\sum_{n=0}^{\infty} a_n = a_0+a_1+a_2+\cdots = a_0+(a_1+a_2+\cdots)=a$$

FIND THE POWER SERIES FOR $xf''(x)-f'(x)+2f(x)$

$$= x\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n-\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n+2\sum_{n=0}^{\infty}a_nx^n$$

$$= \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^{n+1}-\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n+\sum_{n=0}^{\infty}2a_nx^n$$

SHIFT DOWN BY 1

SHIFT UP BY 1

$$= \sum_{n=1}^{\infty}(n+1)(n)a_{n+1}x^n-\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n+\sum_{n=0}^{\infty}2a_nx^n$$

$$= \sum_{n=1}^{\infty}(n+1)na_{n+1}x^n-\left[1\cdot a_1\cdot x^0+\sum_{n=1}^{\infty}(n+1)a_{n+1}x^n\right]+\left[2a_0x^0+\sum_{n=1}^{\infty}\right.$$

$$= -a_1+2a_0+\sum_{n=1}^{\infty}\left((n+1)na_{n+1}-(n+1)a_{n+1}+2a_n\right)x^n$$

$$= \underline{-a_1+2a_0}+\sum_{n=1}^{\infty}\left((n+1)(n-1)a_{n+1}+2a_n\right)x^n$$

NOTE! IF $n=0$, TERM $=1(-1)a_1+2a_0=-a_1+2a_0$

$$= \sum_{n=0}^{\infty}\left((n+1)(n-1)a_{n+1}+2a_n\right)x^n$$

6.2 SOLUTIONS BY POWER SERIES

$y'' + y = 0$

E+U SAYS THERE IS A UNIQUE SOL'N FOR ALL IVP'S BASED ON TH

$y = \sum_{n=0}^{\infty} a_n x^n$

$r^2 + 1 = 0 \longrightarrow r = \pm i$

$y = C_1 \cos x + C_2 \sin x$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \Big(\underbrace{(n+2)(n+1) a_{n+2} + a_n}_{=0 \text{ FOR ALL } n \geq 0} \Big) x^n = 0$$

$(n+2)(n+1) a_{n+2} + a_n = 0$

IF $n \geq 0$ $\quad a_{n+2} = -\frac{1}{(n+2)(n+1)} a_n \longleftarrow$ RECURRENCE RELATIO

LET $a_0 = a_0$

$a_1 = a_1$

$n=0$: $a_2 = -\frac{1}{2 \cdot 1} a_0 = -\frac{1}{2!} a_0$

$n=1$: $a_3 = -\frac{1}{3 \cdot 2} a_1 = -\frac{1}{3!} a_1$

$n=2$: $a_4 = -\frac{1}{4 \cdot 3} a_2 = -\frac{1}{4 \cdot 3} \cdot -\frac{1}{2 \cdot 1} a_0 = \frac{1}{4 \cdot 3 \cdot 2 \cdot 1} a_0 = \frac{1}{4!} a_0$

$n=3: a_5 = -\frac{1}{5\cdot4}a_3 = -\frac{1}{5\cdot4}\cdot -\frac{1}{3\cdot2}a_1 = \frac{1}{5\cdot4\cdot3\cdot2}a_1 = \frac{1}{5!}a_1$

$n=4: a_6 = -\frac{1}{6\cdot5}a_4 = -\frac{1}{6\cdot5}\frac{1}{4\cdot3\cdot2\cdot1}a_0 = -\frac{1}{6!}a_0$

$n=5: a_7 = -\frac{1}{7\cdot6}a_5 = -\frac{1}{7\cdot6}\frac{1}{5\cdot4\cdot3\cdot2}a_1 = -\frac{1}{7!}a_1$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$= a_0 + a_1 x + a_2 x^2 + \cdots$$

$$= a_0 + a_1 x - \frac{1}{2!}a_0 x^2 - \frac{1}{3!}a_1 x^3 + \frac{1}{4!}a_0 x^4 + \frac{1}{5!}a_1 x^5 \ldots$$

$$= a_0 \underbrace{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)}_{\cos x} + a_1 \underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)}_{\sin x}$$

$$= a_0 \cos x + a_1 \sin x$$

IF 2ND ORDER HOMOGEN LDE

ANOTHER METHOD

LET $a_0 = 1,\ a_1 = 0$ + GET A SOL'N

LET $a_0 = 0,\ a_1 = 1$ + GET ANOTHER SOL'N

} AUTOMATICALLY LINEARLY INDEPENDENT

SOLVE

$$y''+x^2y=0$$

$$y=\sum_{n=0}^{\infty} a_n x^n$$

$$y''=\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n$$

$$y''+x^2y=\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n + x^2\sum_{n=0}^{\infty} a_n x^n$$

$$= \qquad \text{''} \qquad + \sum_{n=0}^{\infty} a_n x^{n+2}$$

$$= \qquad \text{''} \qquad + \sum_{n=2}^{\infty} a_{n-2}x^n$$

$$= 2\cdot 1\cdot a_2x^0 + 3\cdot 2\cdot a_3 x^1 + \sum_{n=2}^{\infty} \left((n+2)(n+1)a_{n+2}+a_{n-2}\right)x^n =$$

FOR $n\geq 2$, $(n+2)(n+1)a_{n+2}+a_{n-2}=0$